GTOC X, The 10th Global Trajectory Optimisation Competition
– Settlers of the Galaxy –

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1 The Scenario

In about ten thousand years from the present, humanity will reset its counting of years to zero. Year Zero will be the year when humanity decides the time is ripe for the human race to boldly venture into the galaxy and settle other star systems. One hundred thousand star systems in the galaxy have been identified as being suitable for settlement. Even in this Year Zero, although technologies and knowledge have dramatically progressed, we are still subject to the tyranny of inertia and remain far from the near-instantaneous space travel depicted fancifully in science fiction. However, enormous strides have been made in the ability to live in space, so much so that self-reliant settler vessels can travel though space for hundreds of thousands of generations, making it possible for humans to reach and settle other star systems. The task in GTOC X is to settle as many of the one hundred thousand star systems as possible, in as uniform a spatial distribution as possible, while using as little propulsive velocity change as possible. The settlement of the galaxy starts by fanning out from our home star, Sol. Once another star is settled, further settlements can fan out from that star. Solutions that are submitted earlier rather than later in the submission period will be rewarded a bonus stemming from the fact that humanity's resources are dwindling and the sooner we decide on a settlement plan for the galaxy, the better.

2 The Problem

The stars that are available for settlement are defined in the file `stars.txt`. They are all in circular orbits around the galactic center; their ephemerides are described in the "Equations and Ephemerides" section. The initial settlers from Sol depart in up to three **Mother Ships** and up to two **Fast Ships**. All departures from Sol must occur no later than 10 Myr (10 million years) past Year Zero. Mother Ships each carry up to 10 **Settlement Pods**; a Settlement Pod is released at the instant when the Mother Ship flies by a star (*i.e.,* matches the position of the star). The Settlement Pod's velocity is equal to the Mother Ship's velocity at the instant of release, whereupon the Settlement Pod immediately performs a maneuver to match the velocity of the star, thereby settling that star. The Mother Ship's velocity is unaffected by the flyby (*i.e.,* there is no gravity assist, the star is a massless, moving point in space). The Fast Ships can each rendezvous with a single star (*i.e.,* match its position and velocity), which is then considered settled. Up to three **Settler Ships** can depart from each settled star, as long as at least 2 Myr have elapsed since the star became settled. Each Settler Ship can rendezvous with a single star, which is then considered settled. Stars can only be settled once.

The motion of the ships is subject to the central-force law described in the "Equations and Ephemerides" section. The ships and pods execute impulsive maneuvers to control their motion, as described in the "Propulsive Maneuvers" section. Settlements can be made up to 90 Myr past Year Zero. The task of the competition is to design the trajectories of these ships (*i.e.,* a settlement tree) to maximize the merit function described in section "Merit Function".

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2.1 Equations and Ephemerides

A central-force law governs the motion of both the stars and the ships. The ephemerides of the stars are available analytically because they are assumed to follow circular orbits. The central-force law assumed for the competition approximately models the circular motion observed for actual stars in our Milky Way. Specifically, the circular orbit speed, v_c, for a body at radius r from the galactic center, and the acceleration, f, directed towards the galactic center are

$$v_c(r) = \frac{1}{k_8 r^8 + k_7 r^7 + k_6 r^6 + k_5 r^5 + k_4 r^4 + k_3 r^3 + k_2 r^2 + k_1 r + k_0} \tag{1}$$

$$f(r) = \frac{v_c^2}{r} \tag{2}$$

where the k_i $(i = 0, \ldots, 8)$ are constants defined in the "Constants and Conversions" section. The Cartesian position coordinates of any body (x, y, z) obey the differential equations with respect to time, t:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{x}{r} f(r) \tag{3}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = -\frac{y}{r} f(r) \tag{4}$$

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = -\frac{z}{r} f(r) \tag{5}$$

where $r = \sqrt{x^2 + y^2 + z^2}$.

The motion of the stars also obeys these differential equations, but, due to their orbits being circular, the motion can be expressed analytically and so the following equations should be used to determine position and velocity for the stars as a function of time past Year Zero, t:

$$n \equiv v_c(R)/R \tag{6}$$

$$x = R[\cos(nt + \phi)\cos\Omega - \sin(nt + \phi)\cos i \sin\Omega] \tag{7}$$

$$y = R[\cos(nt + \phi)\sin\Omega + \sin(nt + \phi)\cos i \cos\Omega] \tag{8}$$

$$z = R[\sin(nt + \phi)\sin i] \tag{9}$$

$$v_x = v_c(R)[-\sin(nt + \phi)\cos\Omega - \cos(nt + \phi)\cos i \sin\Omega] \tag{10}$$

$$v_y = v_c(R)[-\sin(nt + \phi)\sin\Omega + \cos(nt + \phi)\cos i \cos\Omega] \tag{11}$$

$$v_z = v_c(R)[\cos(nt + \phi)\sin i] \tag{12}$$

where the values of R, i, Ω, ϕ for each star are specified in the file `stars.txt` which has one header line followed by one line for each star, including Sol. Stars are assigned integer identifiers (IDs) in the file, starting with 0 for Sol and progressing to 100,000. The final column in the file, given for convenience, is the final polar angle of the star which is a derived quantity: $\theta_f = \mathrm{atan2}(y(t_f), x(t_f))$, $t_f = 90$ Myr. It should be noted that we are following common galactic conventions: We take the $+z$ direction as the Galactic North direction and so the stars are orbiting in a retrograde direction around this axis (clockwise when looking down on the galaxy from Galactic North); that is why, for the above equations, the inclinations, i, listed in the file approach 180 degrees.

2.2 Propulsive Maneuvers

Propulsive maneuvers are impulsive, meaning that the velocity of the ship (or pod) changes instantaneously when the impulse is applied. The magnitude of the change in velocity due to an impulse is denoted 'ΔV'. The limits on the magnitude and number of ΔVs are listed in Table 1.

In addition, there is a timing constraint on the maneuvers. For a given ship, consecutive maneuvers must be spaced at least 1 Myr apart in time. A Settlement Pod's rendezvous impulse must also be separated by at least 1 Myr from any impulses performed by the Mother Ship or by any other Settlement Pod from the same Mother Ship. Figure 1 provides a schematic view of a settlement tree starting from Sol.

Table 1: **Propulsive Maneuver Limits**

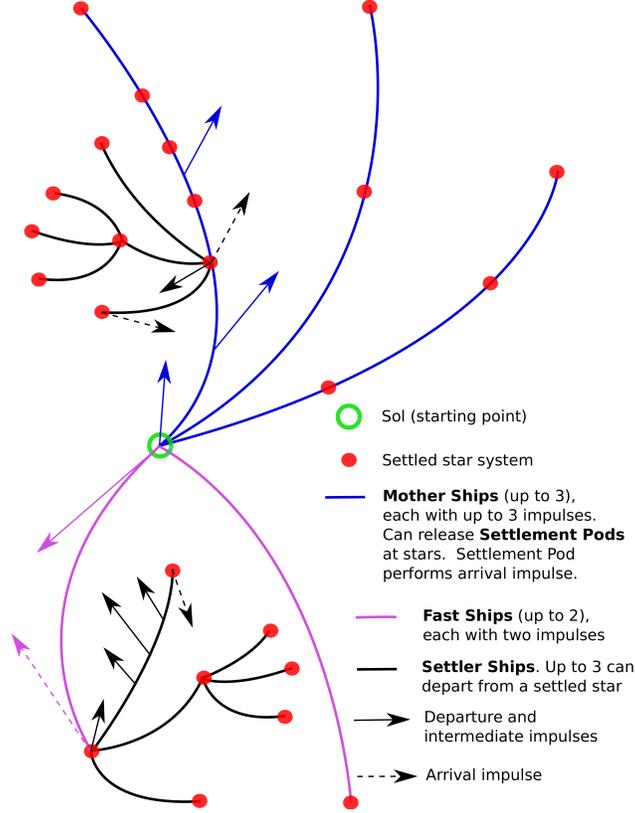| Type of vessel	Available Number of Impulses	Maximum single impulse	Maximum Total ΔV
Mother Ship	3	200 km/s	500 km/s
Settlement Pod	1	300 km/s	300 km/s
Fast Ship	2	1500 km/s	1500 km/s
Settler Ship	5	175 km/s	400 km/s

Figure 1: Schematic of a settlement tree fanning out from Sol. Only some impulses are shown: Three for one of the Mother Ships, two for one of the Fast Ships, one for a Settlement Pod, two for one Settler Ship, and five for another Settler Ship.

2.3 Other Constraints and Tolerances

Ships must not venture closer than 2 kpc to the galactic center, nor further than 32 kpc:

$$2\,\text{kpc} \le \text{r(t)} \le 32\,\text{kpc}, \quad \forall t \tag{13}$$

Upon submission via the competition website, solutions will be checked against our independent propagations generated from the submitted data. Solutions will have to meet the following tolerances to be considered valid:

- Violation of limits on magnitudes of individual ΔVs and Total ΔV per vessel: 0.01 km/s.

- Error in magnitude of arrival impulse: 0.01 km/s.

- Violation of maneuver timing limits and 90 Myr time limit: 1 yr.

- Error in position of vessel with respect to star at rendezvous epoch: 10^{-4} kpc.

- Violation of range limits (Eq. 13): 0.01 kpc.

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2.4 Merit Function

We define first a number of auxiliary equations. Let N be the number of settled stars. Sol does not count as a settled star. First we define the function f of a variable x and parameters μ, s:

$$f(x\,;\,\mu, s) = \begin{cases} 0, & |x - \mu| \geq s \\ \frac{1}{s} - \frac{|x-\mu|}{s^2}, & \text{otherwise.} \end{cases} \tag{14}$$

For the radial and angular coordinates, we then define:

$$f_r(r) = \frac{1}{N} \sum_{i=1}^{N} f(r\,;\, r_i, s_r) \tag{15}$$

$$f_\theta(\theta) = \frac{1}{N} \sum_{i=1}^{N} f(\theta\,;\, \theta_i, s_\theta) \tag{16}$$

where $s_r = 1.0$ kpc, $s_\theta = 2\pi/32$ rad, r_i is the orbital radius of the ith settled star, and θ_i is the final polar angle, θ_f, of the ith settled star (as given in the `stars.txt` file).

Additionally, with $R_{\min} = 2\,\text{kpc}$ and $R_{\max} = 32\,\text{kpc}$, we define the goal functions and parameters:

$$g_r(r) = \alpha(r)\frac{2r}{R_{\max}^2 - R_{\min}^2} \tag{17}$$

$$g_\theta(\theta) = \beta(\theta)\frac{1}{2\pi} \tag{18}$$

$$R_k = k + 2\,\text{kpc}, \qquad 0 \leq k \leq 30 \tag{19}$$

$$\Theta_k = -\pi + 2\pi\frac{k}{32}\,\text{rad}, \quad 0 \leq k \leq 32 \tag{20}$$

where the scaling functions α and β are

$$\alpha(r) = \begin{cases} 0.5833, & r = 2\text{ kpc} \\ 0.4948, & r = 32\text{ kpc} \\ 1, & \text{otherwise.} \end{cases} \tag{21}$$

$$\beta(\theta) = \begin{cases} 0.5, & \theta = -\pi\text{ rad} \\ 0.5, & \theta = \pi\text{ rad} \\ 1, & \text{otherwise.} \end{cases} \tag{22}$$

The following error functions can then be defined:

$$E_r = \sum_{k=0}^{30} \left(\frac{f_r(R_k)}{g_r(R_k)} - 1\right)^2 \tag{23}$$

$$E_\theta = \sum_{k=0}^{32} \left(\frac{f_\theta(\Theta_k)}{g_\theta(\Theta_k)} - 1\right)^2 \tag{24}$$

A bonus factor for submitting a solution early is also defined as:

$$B = 1 + \left(\frac{t_{end} - t_{submission}}{t_{end} - t_{start}}\right)^4, \tag{25}$$

where t_{start} and t_{end} are the start and end epochs of the GTOC X solution submission timeframe, and $t_{submission}$ is the epoch at which the solution file is submitted and received at the GTOC X website.

Finally, the merit function, J, to be maximized is the following:

$$J = B \left(\frac{N}{1 + 10^{-4} \cdot N(E_r + E_\theta)}\right)\left(\frac{\Delta V_{\max}}{\Delta V_{\text{used}}}\right), \tag{26}$$

where ΔV_{used} is the total ΔV used in the settlement tree (the sum of all impulses performed by each ship and pod) and $\Delta V_{\max}$ is maximum ΔV permitted, obtained by summing each vessel's Maximum Total ΔV shown in Table 1. Should there be a tie to five significant figures in J, the tree with larger N will win. Should there still be a tie, the solution with the smaller ΔV_{used} will win.

2.5 Constants and Conversions

The values for the necessary constants and unit conversions are shown in Table 2. Two of the less customary units used, at least in the field of spacecraft trajectory design, are Myr (million years) and kpc (kiloparsecs).

Table 2: **Constants and unit conversions**

Constant	Value	Units
k_8	-1.94316e-12	$(km/s)^{-1}/kpc^8$
k_7	3.7516e-10	$(km/s)^{-1}/kpc^7$
k_6	-2.70559e-08	$(km/s)^{-1}/kpc^6$
k_5	9.70521e-07	$(km/s)^{-1}/kpc^5$
k_4	-1.88428e-05	$(km/s)^{-1}/kpc^4$
k_3	0.000198502	$(km/s)^{-1}/kpc^3$
k_2	-0.0010625	$(km/s)^{-1}/kpc^2$
k_1	0.0023821	$(km/s)^{-1}/kpc$
k_0	0.00287729	$(km/s)^{-1}$
1 kpc	30856775814671900	km
1 Myr	10^6	yr
1 yr	31557600	s

3 Submission Procedures

Solutions are to be submitted via the competition website, `https://gtocx.jpl.nasa.gov/`, by the registered user(s) for each team. In the event of technical difficulties with the website, teams may also submit solutions by email to `gtocx@jpl.nasa.gov` for manual verification and scoring. The file format for the submissions and more detailed information on the submission process are defined in a separate file, `gtocX_submission_format.pdf`, available on the submission website.

4 Acknowledgements